Topic-Sensitive Two-Dimensional Truthmaker Semantics*

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Written: April 8, 2016, Revised: September 20, 2022

Abstract

This paper endeavors to establish foundations for the interaction between hyperintensional semantics and two-dimensional indexing. I examine the significance of the semantics, by developing three, novel interpretations of the framework. The first interpretation provides a characterization of the distinction between fundamental and derivative truths. The interaction between the hyperintensional truthmaker semantics and modal ontology is further examined. The second interpretation demonstrates how the elements of decision theory are definable within the semantics, and provides a novel account of the interaction between probability measures and hyperintensional grounds. The third interpretation concerns the contents of the types of intentional action, and the semantics is shown to resolve a puzzle concerning the role of intention in action. Two-dimensional truthmaker semantics can be interpreted epistemically and metasemantically as well.

1 Introduction

Philosophical applications of two-dimensional intensional semantics have demonstrated that an account of representation which is sensitive to an array of parameters can play a crucial role in explaining the values of linguistic expressions (Kamp, 1967; Kaplan, 1979); the role of speech acts in affecting shared contexts of information (Stalnaker, 1978; Lewis, 1980,a/1998; MacFarlane, 2005); the relationship between conceivability and metaphysical possibility (Chalmers, 1996; 2006, 2008); and the viability of modal realism (Russell, 2010).

In order to circumvent issues for the modal analysis of counterfactuals (2012,a, 2012,b), and to account for the general notion of aboutness and a subject matter (2015), a hyperintensional, 'truthmaker' semantics has recently been developed by Fine (2017a, 2017b). In this essay, I examine the status of two-dimensional indexing in truthmaker semantics, and specify the two-dimensional profile of the grounds for the truth of a formula (Section **2.2**). I proceed, then, to outline

*Acknowledgements: Thanks to Francesco Berto for detailed and very helpful comments.

three novel interpretations of the two-dimensional, hyperintensional framework, beyond the interpretations of multiply indexed intensional semantics that are noted above. The first interpretation provides a formal setting in which to define the distinction between fundamental and derivative truths (Section **3.1**). The second interpretation concerns the interaction between the two-dimensional profile of the verifiers for a proposition, subjective probability, and decision theory (Section **3.2**). Finally, a third interpretation of the two-dimensional hyperintensional framework concerns the types of intentional action. I demonstrate, in particular, how multiply indexed truthmaker semantics is able to resolve a puzzle concerning the role of intention in action (Section **3.3**). Section **4** provides concluding remarks.

2 Two-dimensional Truthmaker Semantics

Two-dimensional semantics provides a framework for regimenting the thought that the value of a formula relative to one parameter determines the value of the formula relative to another parameter. The semantics assigns truth-conditions to formulas, and semantic values to the formula's component terms. The conditions of the formulas and the values of their component terms are assigned relative to the array of intensional parameters. So, e.g., a term may be defined relative to a context; and the value of the term relative to the context will determine the value of the term relative to an index.

Primary, secondary, and 2D intensions can be defined as follows:[1]

- Primary Intension:

  $\texttt{pri}(x) = \lambda c.[\![x]\!]^{c,c}$.

  (The intension is a function mapping formulas, relative to two parameters ranging over possibilities from a first space, to truth-values.);

- Secondary Intension:

  $\texttt{sec}_{v_@}(x) = \lambda w.[\![x]\!]^{v@,w}$.

  (The intension is a function mapping formulas, relative to two parameters, where the first ranges over worlds, one of which is designated as actual, which determines the value of the formula relative to the second parameter ranging over worlds from a distinct space. The secondary intension picks out the semantic value of the formula relative to the second parameter.);

- 2D-Intension:

  $\texttt{2D}(x) = \lambda c \lambda w [\![\mathrm{x}]\!]^{c,w} = 1$.

  (The intension determines a semantic value relative to two parameters, the first ranges over worlds from a first space and the second ranges over worlds from a distinct, second space. The value of the formula relative

[1] The notation for intensions follows the presentation in Chalmers and Rabern (2014: 211-212) and von Fintel and Heim (2011).

to the first parameter determines the value of the formula relative to the second.)

With regard to interpretations of the foregoing, according to Kaplan (1979), an utterance's character is a mapping from the utterance's context of evaluation to the utterance's content. According to Stalnaker (op. cit.; 2004), having distinct functions associated with the value of an utterance provides one means of reconciling the necessity of a formula presupposed by speakers with the contingency of the values of assertions made about that formula.

According to Chalmers (op. cit.), there are cases in which the value of a formula relative to a first parameter, which ranges over epistemically possible worlds, determines the value of a formula relative to a second parameter, which ranges over metaphysically possible worlds. The dependence is recorded by 2D-intensions. Chalmers (2006: 102) provides a conditional analysis of 2D-intensions to characterize the dependence: 'Here, in effect, a term's subjunctive intension depends on which epistemic possibility turns out to be actual. / This can be seen as a mapping from scenarios to subjunctive intensions, or equivalently as a mapping from (scenario, world) pairs to extensions. We can say: the two-dimensional intension of a statement S is true at (V, W) if V verifies the claim that W satisfies S. If $[A]_1$ and $[A]_2$ are canonical descriptions of V and W, we say that the two-dimensional intension is true at (V, W) if $[A]_1$ epistemically necessitates that $[A]_2$ subjunctively necessitates S. A good heuristic here is to ask "If $[A]_1$ is the case, then if $[A]_2$ had been the case, would S have been the case?". Formally, we can say that the two-dimensional intension is true at (V, W) iff '$\Box_1([A]_1 \rightarrow \Box_2([A]_2 \rightarrow S))$' is true, where '$\Box_1$' and '$\Box_2$' express epistemic and subjunctive necessity respectively'. Epistemic possibility entails metaphysical possibility in cases in which formulas are, furthermore, 'super-rigid' (2012: 474), i.e. have a 'constant two-dimensional intension (370), i.,e. map to the same truth-value in all epistemically possible worlds and all metaphysically possible worlds (369).

According to Lewis (op. cit.), the context may be treated as a concrete situation ranging over individuals, times, locations, and worlds; and the index may be treated as ranging over shiftable parameters of the context. According to MacFarlane (op. cit.), formulas may receive their value relative to a context ranging over two distinct agents; the context determines the value of an index ranging over their states of information; and the value of the formula may yet be defined relative to a third parameter ranging over the states of an independent, third assessor. Finally, in decision theory, the value of a formula relative to a context, which ranges over a time, location, and agent, constrains the value of the formula relative to a first index on which a space of the agent's possible acts is built, and the latter will subsequently constrain the value of the formula relative to a second index on which a space of possible outcomes may be built.

2.1 Truthmaker Semantics

A hyperintensional, 'truthmaker' semantics has recently been developed by Fine (2017a, 2017b).[2] Truthmaker semantics has been applied, in order to explain the conditions under which parts of worlds, rather than worlds in their entirety, verify propositions.

Truthmaker semantics is defined over a state space, F = $\langle$S, $\sqsubseteq\rangle$, where S is a set of states which are parts of a world, and $\sqsubseteq$ is a parthood relation on S which is a partial order, such that it is reflexive (x $\sqsubseteq$ x), anti-symmetric [(if x $\sqsubseteq$ y) $\wedge$ (y $\sqsubseteq$ x), then x = y], and transitive (x $\sqsubseteq$ y, y $\sqsubseteq$ z; x $\sqsubseteq$ z) (2017a: 19).

A proposition P $\subseteq$ S is verifiable if P is non-empty, and is otherwise unverifiable (20).

Following Fine (2021), fusions of states, x $\sqcup$ y, are always defined.

A model, M, over F is a tuple, M = $\langle$F,D,V$\rangle$, where D is a domain of closed formulas (i.e. propositions), and V is an assignment function mapping propositions P$\in$D to pairs of subsets of S, {1,0}, i.e. the verifier and falsifier of P, such that $[\![P]\!]^{+} = 1$ and $[\![P]\!]^{-} = 0$ (35).

The verification-rules in truthmaker semantics are then the following:

s $\vdash$ P if s$\in[\![P]\!]^{+}$

(s verifies P, if s is a truthmaker for P i.e. if s is in P's extension);

s $\dashv$ P if s$\in[\![P]\!]^{-}$

(s falsifies P, if s is a falsifier for P i.e. if s is in P's anti-extension);

s $\vdash$ $\neg$P if s $\dashv$ P

(s verifies not P, if s falsifies P);

s $\dashv$ $\neg$P if s $\vdash$ P

(s falsifies not P, if s verifies P);

s $\vdash$ P $\wedge$ Q if $\exists$t,u, t $\vdash$ P, u $\vdash$ Q, and s = t $\sqcup$ u

(s verifies P and Q, if s is the fusion of states, t and u, t verifies P, and u verifies Q);

s $\dashv$ P $\wedge$ Q if s $\dashv$ P or s $\dashv$ Q

(s falsifies P and Q, if s falsifies P or s falsifies Q);

s $\vdash$ P $\vee$ Q if s $\vdash$ P or s $\vdash$ Q

(s verifies P or Q, if s verifies P or s verifies Q);

s $\dashv$ P $\vee$ Q if $\exists$t,u, t $\dashv$ P, u $\dashv$ Q, and s = t $\sqcup$ u

(s falsifies P or Q, if s is the state overlapping the states, t and u, t falsifies P, and u falsifies Q);

s $\vdash$ $\forall$xϕ(x) if $\exists$s$_1$, ..., s$_n$, with s$_1$ $\vdash$ ϕ(a$_1$), ..., s$_n$ $\vdash$ ϕ(a$_n$), and s = s$_1$ $\sqcup$ ... $\sqcup$ s$_n$

[s verifies $\forall$xϕ(x) "if it is the fusion of verifiers of its instances ϕ(a$_1$), ..., ϕ(a$_n$)" (Fine, 2017c)];

s $\dashv$ $\forall$xϕ(x) if s $\dashv$ ϕ(a) for some individual a in a domain of individuals (op. cit.)

[s falsifies $\forall$xϕ(x) "if it falsifies one of its instances" (op. cit.)];

[2] The logic for the semantics is classical. Fine (2014) develops a truthmaker semantics for intuitionistic logic.

$s \vdash \exists x\phi(x)$ if $s \vdash \phi(a)$ for some individual a in a domain of individuals (op. cit.)

[s verifies $\exists x\phi(x)$ "if it verifies one of its instances $\phi(a_1), \ldots, \phi(a_n$" (op. cit.)];

$s \dashv \exists x\phi(x)$ if $\exists s_1, \ldots, s_n$, with $s_1 \dashv \phi(a_1), \ldots, s_n \dashv \phi(a_n)$, and $s = s_1 \sqcup \ldots \sqcup s_n$ (op. cit.)

[s falsifies $\exists x\phi(x)$ "if it is the fusion of falsifiers of its instances" (op. cit.)];

s exactly verifies P if and only if $s \vdash P$ if $s \in [\![P]\!]$;

s inexactly verifies P if and only if $s \triangleright P$ if $\exists s' \leq S$, $s' \vdash P$; and

s loosely verifies p if and only if, $\forall t$, s.t. $s \sqcup t$, $s \sqcup t \vdash p$, where $\sqcup$ is the relation of compatibility (35-36);

Differentiated contents may be defined as follows.[3] A state $s \sqsubseteq S$ is differentiated only if s is the fusion of distinct parts, s.t. $s = s_1 \sqcup s_2$. There is thus an initial state, s_1; an additional state, s_2; and a total state, s. The three states correspond accordingly to three contents: The initial content $s_1 \vdash P_1$; the additional content, $s_2 \vdash P_2$; and the total content, $s \vdash P_{1,2}$ (2017b: 15).

Finally, subject matters may be defined as follows.

A verifiable proposition, $[\![P]\!]^+$, is about a positive subject matter, $\mathbf{p}^+$ (20-21).

A falsifiable proposition, $[\![P]\!]^-$ is about a negative subject matter, $\mathbf{p}^-$ (21).

The intersection of the subject matters both verified and falsified by the fusion of a number of states comprise a comprehensive subject matter:

$\mathbf{p}_{1,+,-} = \mathbf{p}_{1,+} \sqcap \mathbf{p}_{1,-} = \langle s \vdash P \text{ and } s \dashv P \rangle$;

$\mathbf{p}_{2,+,-} = \mathbf{p}_{2,+} \sqcap \mathbf{p}_{2,-} = \langle s \vdash P_2 \text{ and } s \dashv P_2 \rangle$; such that,

$\mathbf{p}_{1,2,+,-} = \mathbf{p}_{1,2,+} \sqcap \mathbf{p}_{1,2,-} = \langle s \vdash P_{1,2} \text{ and } s \dashv P_{1,2} \rangle$ (op. cit.).

The union of the subject matters that are either verified or falsified by the fusion of a number of states comprise a differentiated subject matter:

$\mathbf{p}_{1,+/-} = \mathbf{p}_{1,+} \sqcup \mathbf{p}_{1,-} = \langle s \vdash P \text{ or } s \dashv P \rangle$;

$\mathbf{p}_{2,+,-} = \mathbf{p}_{2,+} \sqcup \mathbf{p}_{2,-} = \langle s \vdash P_2 \text{ or } s \dashv P_2 \rangle$; such that,

$\mathbf{p}_{1,2,+/-} = \mathbf{p}_{1,2,+} \sqcup \mathbf{p}_{1,2,-} = \langle s \vdash P_{1,2} \text{ or } s \dashv P_{1,2} \rangle$ (op. cit.).

Informally, propositions P and Q are about the same subject matters, **p** and **q**, when the following conditions hold:

P is exactly about Q if $\mathbf{p} = \mathbf{q}$;

P is partly about Q if **p** and **q** overlap, such that $\exists u \sqsubset S(u \vdash R)$; $\forall s_1, s_2 \sqsubseteq S$, $s_1 \vdash P$, $s_2 \vdash Q$; and $u = s_1 \sqcap s_2$, such that $R = P \cap Q$;

P is entirely about Q if $\mathbf{p} \subseteq \mathbf{q}$; and

P is about Q in its entirety if $\mathbf{p} \supseteq \mathbf{q}$ (5).

2.2 Two-dimensional Truthmaker Semantics

In order to account for two-dimensional indexing, we augment the model, M, with a second state space, S*, on which we define both a new parthood relation, $\sqsubset^*$, and partial function, V*, which serves to map propositions in D to pairs of subsets of S*, {1,0}, i.e. the verifier and falsifier of P, such that $[\![P]\!]^+ = 1$

[3] Fine (op. cit.: 8, 12) avails of product spaces in his discussion of content and subject matter, though we continue here to work with a single space for ease of exposition.

and $[\![P]\!]^- = 0$. Thus, M = ⟨S, S*, D, ⊏, ⊏*, V, V*⟩. The two-dimensional hyperintensional profile of propositions may then be recorded by defining the value of P relative to two parameters, c,i: c ranges over subsets of S, and i ranges over subsets of S*.

(*) M,s∈S,s*∈S* ⊢ P iff:
(i) $\exists c_s[\![P]\!]^{c,c} = 1$ if s∈$[\![P]\!]^+$; and
(ii) $\exists i_{s*}[\![P]\!]^{c,i} = 1$ if s*∈$[\![P]\!]^+$

(Distinct states, s,s*, from distinct state spaces, S,S*, provide a multi-dimensional verification for a proposition, P, if the value of P is provided a truthmaker by s. The value of P as verified by s determines the value of P as verified by s*).

We say that P is hyper-rigid iff:

(**) M,s∈S,s*∈S* ⊢ P iff:
(i) $\forall c'_s[\![P]\!]^{c,c'} = 1$ if s∈$[\![P]\!]^+$; and
(ii) $\forall i_{s*}[\![P]\!]^{c,i} = 1$ if s*∈$[\![P]\!]^+$
Hyper-rigidity is the analogue of super-rigidity in the hyperintensional setting.

The foregoing provides a two-dimensional hyperintensional semantic framework within which to interpret the values of a proposition. Two-dimensional truthmakers can further be exact, inexact, or loose:

s is a two-dimensional exact truthmaker of P if and only if (*);

s is a two-dimensional inexact truthmaker of P if and only if ∃s'⊏S, s→ s', s' ⊢ P and such that

$\exists c_{s'}[\![P]\!]^{c,c} = 1$ if s'∈$[\![P]\!]^+$, and

$\exists i_{s*}[\![P]\!]^{c,i} = 1$ if s*∈$[\![P]\!]^{+}$[4];

s is a two-dimensional loose truthmaker of P if and only if, ∃t, s.t. s ⊔ t, s ⊔ t ⊢ P:

$\exists c_{s\sqcup t}[\![P]\!]^{c,c} = 1$ if s'∈$[\![P]\!]^+$, and

$\exists i_{s*}[\![P]\!]^{c,i} = 1$ if s*∈$[\![P]\!]^+$.

- $[\![P]\!]^{c,i}$ is exactly about $[\![Q]\!]^{c,i}$ if $f_{1-1}[\mathbf{p}^{c,i} \iff \mathbf{q}^{c,i}]$

  (Suppose that the values of P and of Q are multi-dimensionally determined, as above. Then P is exactly about Q if there is a bijection between the multi-dimensionally individuated subject matters that they express);

- $[\![P]\!]^{c,i}$ is partly about $[\![Q]\!]^{c,i}$ if **p** and **q** overlap, s.t. ∃u⊏S, s.t. u ⊢ R, and $\forall s_1,s_2 \sqsubseteq S$, $s_1 \vdash P$, $s_2 \vdash Q$, and $u = s_1 \sqcap s_2$ such that $R^{c,c} = P \cap Q$. A neighborhood function, A, maps u to a state s* in i where s* ⊢ $R^{c,i}$.

- $[\![P]\!]^{c,i}$ is entirely about $[\![Q]\!]^{c,i}$ if $\mathbf{p}^{c,i} \Leftarrow \mathbf{q}^{c,i}$

  (Suppose that the values of P and of Q are multi-dimensionally determined. Then P is entirely about Q if there is a surjection from the subject matter of Q onto the subject matter of P);

[4]'x → x'' is read as claiming that the state, x, is extended by the state, x', while not forming a fusion of states, rather than as entailment or containment.

- $[\![P]\!]^{c,i}$ is about $[\![Q]\!]^{c,i}$ in its entirety if $\mathbf{p}^{c,i} \Rightarrow \mathbf{q}^{c,i}$

  (Suppose that the values of P and of Q are multi-dimensionally determined. Then P is about Q in its entirety if there is an injection from the subject matter of P onto the subject matter of Q).

3 Topic-sensitive Two-dimensional Truthmaker Semantics

I will present topic models first, followed by epistemic two-dimensional truthmaker semantics. I will also outline the clauses for the hyperintensions of a topic-sensitive and possible world as well as truthmaker account of epistemic two-dimensional semantics.

Following the presentation of topic models in Berto (2018; 2019), Canavotto et al (2020), and Berto and Hawke (2021), atomic topics comprising a set of topics, T, record the hyperintensional intentional content of atomic formulas, i.e. what the atomic formulas are about at a hyperintensional level. Topic fusion is a binary operation, such that for all x, y, z∈T, the following properties are satisfied: idempotence (x ⊕ x = x), commutativity (x ⊕ y = y ⊕ x), and associativity [(x ⊕ y) ⊕ z = x ⊕ (y ⊕ z)] (Berto, 2018: 5). Topic parthood is a partial order, ≤, defined as ∀x,y∈T(x ≤ y $\iff$ x ⊕ y = y) (op. cit.: 5-6). Atomic topics are defined as follows: Atom(x) $\iff$ ¬∃y < x, with < a strict order. Topic parthood is thus a partial ordering such that, for all x, y, z∈T, the following properties are satisfied: reflexivity (x ≤ x), antisymmetry (x ≤ y ∧ y ≤ x → x = y), and transitivity (x ≤ y ∧ y ≤ z → x ≤ z) (6). A topic frame can then be defined as {W, R, T, ⊕, t}, with t a function assigning atomic topics to atomic formulas. For formulas, ϕ, atomic formulas, p, q, r (p_1, p_2, ...), and a set of atomic topics, Utϕ = {p_1, ... p_n}, the topic of ϕ, t(ϕ) = ⊕Utϕ = t(p_1) ⊕ ... ⊕ t(p_n) (op. cit.). Topics are hyperintensional, though not as fine-grained as syntax. Thus t(ϕ) = t(¬¬ϕ), tϕ = t(¬ϕ), t($\phi \wedge \psi$) = t(ϕ) ⊕ t(ψ) = t($\phi \vee \psi$) (op. cit.).

The diamond and box operators can then be defined relative to topics:
$\langle$M,w$\rangle \Vdash \diamond^t\phi$ iff $\langle R_{w,t}\rangle(\phi)$
$\langle$M,w$\rangle \Vdash \Box^t\phi$ iff $[R_{w,t}](\phi)$, with
$\langle R_{w,t}\rangle(\phi)$:= {w'∈Wt'∈T | $R_{w,t}$[w', t'] ∩ $\phi \neq \emptyset$ and t'(ϕ) ≤ t(ϕ)
$[R_{w,t}](\phi)$:= {w'∈Wt'∈T | $R_{w,t}$[w', t'] ⊆ ϕ and t'(ϕ) ≤ t(ϕ).
Hyperintensions can then be defined as functions from world, topic pairs to extensions.

- Epistemic Hyperintension:

  $\mathtt{pri}_t(x) = \lambda c\lambda t.[\![x]\!]^{c\cap t, c\cap t}$,

- Subjunctive Hyperintension:

  $\mathtt{sec}_{v_@\cap t}(x) = \lambda w\lambda t.[\![x]\!]^{v_@\cap t, w\cap t}$

- 2D-Hyperintension:
  $\mathtt{2D}(x) = \lambda c\lambda w\lambda \mathrm{t}[\![\mathrm{x}]\!]^{c\cap t,w\cap t} = 1.$

We can also combine topics with truthmakers rather than worlds, thus countenancing doubly hyperintensional semantics, i.e. topic-sensitive epistemic two-dimensional truthmaker semantics:

- Epistemic Hyperintension:
  $\mathtt{pri}_t(x) = \lambda s\lambda \mathrm{t}.[\![x]\!]^{s\cap t,s\cap t}$, with s a truthmaker from an epistemic state space.

- Subjunctive Hyperintension:
  $\mathtt{sec}_{v_@\cap t}(\mathrm{x}) = \lambda w\lambda \mathrm{t}.[\![x]\!]^{v_@\cap t,w\cap t}$, with w a truthmaker from a metaphysical state space.

- 2D-Hyperintension:
  $\mathtt{2D}(x) = \lambda s\lambda w\lambda \mathrm{t}[\![\mathrm{x}]\!]^{s\cap t,w\cap t} = 1.$

4 New Interpretations

The two-dimensional account of truthmaker semantics provides a general framework in which a number of interpretations of the state spaces at issue can be defined. Two-dimensional truthmaker semantics may receive, e.g., the 'metasemantic' interpretation of the two-dimensional framework. The metasemantic interpretation accommodates the update effects of contingently true assertions on a context set with regard to necessary propositions (cf. Stalnaker, op. cit.). The framework may further be provided an 'epistemic' interpretation, in order to countenance hyperintensional distinctions in the relations between conceivability, i.e. the space of an agent's epistemic states, and metaphysical possibility, i.e. the state space of facts (cf. Chalmers, op. cit.).[5] In this section, I advance three novel interpretations of two-dimensional semantics, as witnessed by the new relations induced by the interaction between two-dimensional indexing and hyperintensional value assignments. The three interpretations concern (i) the distinction between fundamental and derivative truths; (ii) probabilistic grounding in the setting of decision theory; and (iii) the structural contents of the types of intentional action.

4.1 Fundamental and Derivative Truths

The first novel interpretation concerns the distinction between fundamental and derivative truths. In the foregoing model, the value of the subject matter expressed by a proposition may be verified by states in a first space, which determine, then, whether the proposition is verified by states in a second space. Allowing the first space to be interpreted so as to range over fundamental facts

[5]Cf. Khudairi (2017) for further discussion.

and the second space to be interpreted so as to range over derivative facts permits a precise characterization of the determination relations between the fundamental and derivative grounds for a truth.

Suppose, e.g., that the fundamental facts concern the computational characterization of a subject's mental states, and let the fundamental facts comprise the first state space. Let the derivative facts concern states which verify whether the subject is consciously aware of their mental representations, and let the derivative facts comprise the second state space. Finally, let ϕ be a psychological formula, e.g. a characterization of a mental state in an experimental task where there is a particular valence for the contrast-level of a stimulus. The formula's having a truthmaker in the first space – where the states of which range, as noted, over the subject's psychofunctional facts – will determine whether the formula has a truthmaker in the second space – where the states of which range over the mental representations of which the subject is consciously aware. If the deployment of some attentional functions provides a necessary condition on the instantiation of phenomenal awareness, then the role of the state of the attentional function in the first space in verifying ϕ will determine whether ϕ is subsequently verified relative to the second space. Intuitively: Attending to a stimulus with a particular value will constrain whether a truthmaker can be provided for being consciously aware of the stimulus. If the computational facts at issue are fundamental, and the phenomenal facts at issue are derivative, then a precise characterization may be provided of the multi-dimensional relations between the verifiers which target fundamental and derivative truths.

4.2 Decision Theory

A second novel interpretation of two-dimensional truthmaker semantics concerns the types of intentional action, and the interaction of the latter with decision theory. As noted in the foregoing, two-dimensional intensional semantics may be availed of in order to explain how the value of a formula relative to a context ranging over an agent and time will determine the value of the formula relative to an index ranging over a space of admissible, actions made on the basis of the formula, where the value of the formula relative to the context and first index will determine the value of the formula relative to a second index, ranging over a space of outcomes.

One notable feature of the decision-theoretic interpretation is that it provides a natural setting in which to provide a gradational account of truthmaking. A proposition and its component expressions are true, just if they are verified by states in a state space, such that the state and its parts fall within the proposition's extension. In decision theory, a subject's expectation that the proposition will occur is recorded by a partial belief function, mapping the proposition to real numbers in the $\{0,1\}$ interval. The subject's desire that the proposition occurs is recorded by a utility function, the quantitative values of which – e.g., 1 or 0 – express the qualitative value of the proposition's occurrence. The evidential expected utility of a proposition's occurrence is calculated as the probability of its obtaining conditional on an agent's action, as multiplied by the utility to the agent of the proposition's occurrence. The causal expected utility of the proposition's occurrence is calculated as the probability of its obtaining, conditional on both the agent's acts and background knowledge of the causal efficacy of their actions, multiplied by the utility of the proposition's occurrence.

There are three points at which a probabilistic construal of the foregoing may be defined. One point concerns the objective probability that the proposition will be verified, i.e. the chance thereof. The second point concerns subjective probability with which a subject partially believes that the proposition will obtain. A third point concerns the probability that an outcome will occur, where the space of admissible outcomes will be constrained by a subject's acts. An agent's actions will, in the third case, constrain the admissible verifiers in the space of outcomes, and thus the probability that the verifier for the proposition will obtain as an outcome.[6]

In order formally to countenance the foregoing, we define a probability measure on a state space, such that the probability measure satisfies the Kolmogorov axioms: normality [$\Pr(T) = 1$]; non-negativity [$\Pr(\phi) \geq 0$]; additivity [For disjoint ϕ and ψ[$\Pr(\phi \cup \psi) = \Pr(\phi) + \Pr(\psi)$]]; and conditionalization [$\Pr(\phi \mid \psi) = \Pr(\phi \cap \psi) / \Pr(\psi)$]. In order to account for the interaction between objective probability and the verification-conditions in truthmaker semantics, we avail, then, of a regularity condition in our earlier model, M, in which the assignment function, V, maps propositions $P \in D$ to pairs of subsets of S, $\{1,0\}$, i.e. the verifier and falsifier of P, such that $[\![P]\!]^+ = \{0,1\}$ and $[\![P]\!]^- = 1 - P$. In our gradational truthmaker semantics, a state, s, verifies a proposition, P, if the probability that s is in P's extension is greater than or equal to .5:

$s \vdash P$ if $\Pr(s \in [\![P]\!]^+) \geq .5$.

A state, s, falsifies a proposition P if the probability that s is in P's extension is less than .5 iff the probability that s is in P's anti-extension is greater than or equal to .5

$s \dashv P$ if $\Pr(s \in [\![P]\!]^-) \geq .5$

iff $\Pr(s \in [\![P]\!]^+) < .5$.

The subjective probability with regard to the proposition's occurrence is expressed by a probability measure satisfying the Kolmogorov axioms as defined on a second state space, i.e., a space whose points are interpreted as concerning the subject's states of information. The formal clauses for partial belief in truthmaker semantics are the same as in the foregoing, save that the probability

[6] A proponent of metaphysical indeterminacy might further suggest that the verifiers are themselves gradational; thus, rather than target the probability of a verifier's realization, the proponent of metaphysical indeterminacy will suggest that a proposition P is made true only to a certain degree, such that both of the proposition's extension and anti-extension will have non-negative, real values. One objection to the foregoing account of metaphysical indeterminacy for truthmakers is, however, that the metalogic for many-valued logic is classical (cf. Williamson, 2014). A distinct approach to metaphysical indeterminacy is proffered by Barnes and Williams (2011), who argue that metaphysical indeterminacy consists in persistently unpointed models, i.e. a case in which it is unclear which among a set of worlds is actual, even upon filtering the set with precisifications. A proponent of metaphysical indeterminacy for probabilistic truthmaker semantics might then argue both that the realization of a verifier has a gradational value and that it is indeterminate which of the states which can verify a given formula is actual.

measures express the mental states of an agent, by being defined on the space of their states of information.

Finally, the interaction between objective and subjective probability measures in hyperintensional semantics may be captured as follows.

One way to countenance the foregoing is via the interaction between the chance of a proposition's occurrence, the subject's partial belief that the proposition will occur, and the spaces for the subjects actions and outcomes. The formal clause for the foregoing will then be as follows:

M,s $\vdash$ $[\![\mathrm{P}]\!]^{c(c',a,o)} > .5$,

where c ranges over the space of physical states, and a probability measure recording objective chance is defined thereon; c' ranges over the space of an agent's states of information, and the value of P relative to c' determines the value of P relative to the space of the agent's acts, a, where the latter determines the space of admissible outcomes concerning P's occurrence, o. Thus, the parameters, c',a,o possess a hyperintensional two-dimensional profile, and the space of physical states, c, determines the values of the subject's partial beliefs and their subsequently conceivable actions and outcomes.

Accounting for the relation between c and c' – i.e., specifying a norm on the relation between chances and credences – provides one means by which to account for how objective gradational truthmakers interact with a subject's partial beliefs about whether propositions are verified. Following Lewis (1980,b/1987), a candidate chance-credence norm may be what he refers to as the 'principal principle'.[7] The principal principle states that an agent's partial belief that a proposition will be verified, conditional on the objective chance of the proposition's occurrence and the admissible evidence, will be equal to the objective chance of the proposition's occurrence itself:

$\mathrm{Pr}_s(\mathrm{P} \mid \mathrm{ch}(\mathrm{P}) \wedge \mathrm{E}) = \mathrm{ch}(\mathrm{P})$.

4.3 Intentional Action

A third novel interpretation of two-dimensional hyperintensional semantics provides a natural setting in which to delineate the structural content of the types of intentional action. For example, the mental state of intending to pursue a course of action may be categorized as falling into three types, where intending-that is treated as a two-dimensional hyperintensional state. One type targets a unique structural content for the state of acting intentionally, such that an

[7] See Pettigrew (2012), for a justification of a generalized version of the principal principle based on Joyce's (1998) argument for probabilism. Probabilism provides an accuracy-based account of partial beliefs, defining norms on the accuracy of partial beliefs with reference only to worlds, metric ordering relations, and probability measures thereon. The proposal contrasts to pragmatic approaches, according to which a subject's probability and utility measures are derivable from a representation theorem, only if the agent's preferences with regard to a proposition's occurrence are consistent (cf. Ramsey, 1926). Probabilism states, in particular, that, if there is an ideal subjective probability measure, the ideality of which consists e.g. in its matching objective chance, then one's probability measure ought to satisfy the Kolmogorov axioms, on pain of there always being a distinct probability measure which will be metrically closer to the ideal state than one's own.

agent intends to bring it about that ϕ just if the intention satisfies a clause which mirrors that outlined in the last paragraph:

- $[\![\text{Intenton-in-Action}(\phi)]\!]_w = 1$ only if $\exists$w'$[\![\phi]\!]^{w',c(=t,l),a,o} = 1$.

A second type of intentional action may be recorded by a future-directed state, such that an agent intends to ϕ only if they intend to pursue a course of action in the future, only if there is a state and a future time relative to which the agent's intention is satisfied:

- $[\![\text{Intention-for-the-future}(\phi)]\!]_w = 1$ only if $\exists$w'$\forall$t$\exists$t'[t< t' $\wedge$ $[\![\phi]\!]^{w',t'} = 1$].

Finally, a third type of intentional action concerns reference to the intention as an explanation for one's course of action. Khudairi (op. cit.) regiments the structural content of this type of intention as a state which receives its value only if a hyperintensional grounding operator which takes scope over a proposition and an action, receives a positive semantic value.

- $[\![\text{Intention-with-which}(\phi)]\!]_w = 1$ only if $\exists$w'[$[\![\psi]\!]^{w'} = 1 \wedge [\![G(\phi,\psi)]\!] = 1$],

where G(x,y) is a grounding operator encoding the explanatory connection between ϕ and ψ.

The varieties of subject matter, as defined in two-dimensional truthmaker semantics, can be availed of in order to enrich the present approach. Having multiple state spaces from which to define the verifiers of a proposition enables a novel solution to issues concerning the interaction between action and explanation. The third type of intentional action may be regimented, as noted, by the agent's reference to an intention as an explanation for her course of action.

The foregoing may also be availed of, in order to provide a novel solution to an issue concerning the interaction between involuntary and intentional action. The issue is as follows. Wittgenstein (1953/2009; 621) raises the inquiry: 'When I raise my arm, my arm goes up. Now the problem arises: what is left over if I subtract the fact that my arm goes up from the fact that I raise my arm?' Because the arm's being raised has at least two component states, namely, the arm's going up and whatever the value of the variable state might be, the answer to Wittgenstein's inquiry is presumably that the agent's intentional action is the value of the variable state, such that a combination of one's intentional action and one's arm going up is sufficient for one's raising one's arm. The aforementioned issue with the foregoing concerns how precisely to capture the notion of partial content, which bears on the relevance of the semantics of the component states and the explanation of the unique state entrained by their combination.

Given our two-dimensional truthmaker semantics, a reply to Wittgenstein's inquiry which satisfies the above desiderata may be provided. Let W express a differentiated subject matter, whose total content is that an agent's arm is raised. W expresses the total content that an agent's arm is raised, because W is comprised of an initial content, U (that one's arm goes up), and an additional content, R (that one intends to raise one's arm).

The verifier for W may be interpreted as a two-dimensional loose truthmaker. Let c range over an agent's motor states, S. Let i range over an agent's states of information, S*. We define a state for intentional action in the space of the agent's motor actions. The value of the state is positive just if a selection function, f, is a mapping from the powerset of motor actions in S to a unique state s' in S. This specifies the initial, partial content, U, that one's arm goes up. An intention may then be defined as a unique state, s*, in the agent's state of information, S*. The state, s*, specifies the additional, partial content R, that one intends to raise one's arm.

Formally:

s $\vdash$ U only if $\exists$s'$\sqsubset$S, such that f: s$\rightarrow$ s', s.t. s' $\vdash$ U,

$\exists$s*, s* $\vdash$ R, and

W = U $\sqcup$ R.

The two-dimensional loose truthmaker for one's arm being raised may then be defined as follows:

$\exists c_{s\rightarrow s'}[\![W]\!]^{c,c} = 1$ if s'$\in[\![W]\!]^{+}$, and

$\exists i_{s*}[\![W]\!]^{c,i} = 1$ if s*$\in[\![W]\!]^{+}$.

Intuitively, the value of the total content that one's arm is raised is defined relative to a set of motor states – where a first intentional action selects a series of motor states which partly verify that one's arm goes up. The value of one's arm being raised, relative to (the intentionally modulated) motor state of one's arm possibly going up, determines the value of one's arm being raised relative to the agent's distinct intention to raise their arm. The agent's first intention selects among the admissible motor states, and – all else being equal – the motor states will verify the fact that one's arm goes up.[8] The fusion of (i) the state corresponding to the initial partial content that one's arm goes up, and (ii) the state corresponding to the additional partial content that one intends to raise one's arm, is sufficient for the verification of (iii) the state corresponding to the total content that one's arm is raised.

5 Concluding Remarks

In this essay, I have endeavored to establish foundations for the interaction between two-dimensional indexing and hyperintensional semantics. I examined, then, the philosophical significance of the framework by developing three, novel interpretations of two-dimensional truthmaker semantics, in light of the new relations induced by the model. I noted that two-dimensional truthmaker semantics can be interpreted epistemically and metaphysically as well.

The first interpretation enables a rigorous characterization of the distinction between fundamental and derivative truths. The second interpretation evinces how the elements of decision theory are definable within the two-dimensional

[8]The role of the first intention in acting as a selection function on the space of motor actions corresponds to the comparator functions stipulated in the cognitive science of action theory. For further discussion of the comparator model, see Frith et al. (2000) and Pacherie (2012).

hyperintensional setting, and a novel account was then outlined concerning the interaction between probability measures and hyperintensional grounds. The third interpretation of two-dimensional hyperintensional semantics concerns the structural content of the types of intentional action. Finally, I demonstrated how the hyperintensional array of state spaces, relative to which propositions may be verified, may serve to resolve a previously intransigent issue concerning the role of intention in action.

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